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Theory of Clarifier Operation. IV. Orthokinetic Flocculation in Concentrated Slurries

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Abstract

The operation of a continuous-flow sludge blanket clarifier is simulated by use of the continuity equations. The result of including an orthokinetic flocculation term due to the small-scale turbulence generated by the falling particles is examined and compared with the results obtained when only a "catch-up" flocculation mechanism is included. Turbulence flocculation permits the agglomeration of fines and results in an increase in clarifier efficiency. The effects of hydraulic loading and influent solids volume fraction are studied.

INTRODUCTION

In earlier papers of this series we used the continuity equations to model quiescent settling (*1*), rectangular clarifier operation (*2*), and the operation of three types of upflow clarifiers (*3*). Flocculation, floc disruption, variation of viscosity with solids volume fraction, and departures from Stokes' law were taken into account. The literature is reviewed in the first of these papers. Another significant reference is S. Chang's dissertation on the modeling of clarifier operation (*4*), which provides an excellent review through 1972 as well as a quite rigorous approach to clarifier modeling; his approach makes quite heavy demands on computer size and time. Argaman and Kaufman have published a quite detailed report on orthokinetic flocculation which bears closely on the work we present here (*5*).

In our previous modeling of clarifier operation we employed a "catch-up" mechanism for orthokinetic flocculation in which the larger, faster falling particles catch up to and coalesce with the smaller, more slowly falling particles. We assumed in our model that particle volumes were integral multiples of an elementary particle volume, $V_n = nV_1$. A difficulty with the catch-up mechanism when applied to such distributions of particle sizes is that it provides no means for the coalescence of two particles of the same size. In some simulation runs this led to the accumulation of large volume fractions of 1-particles in the upper section of the clarifier; the resulting increase in viscosity, decrease in density difference between the particles and the slurry, and increase in liquid linear velocity led to discharge of these "fines" in the clarifier overflow. In actual fact, these fines would be expected to undergo orthokinetic flocculation, at least to some degree, and settle down to the sludge wasting point.

Perikinetic flocculation (resulting from Brownian motion) was considered and certainly provides a mechanism for the coagulation of two particles of identical size. It occurs at a significant rate, however, only when the particles are extremely small, and hence is important only during the first few seconds, at most, of floc formation. It seemed unreasonable to assume that our "fines" were anywhere near that small. We therefore sought another mechanism.

As the particles fall down through the liquid in the clarifier, power is dissipated in the liquid. This power dissipation results in the formation of local velocity gradients in the liquid, and these velocity gradients should result in orthokinetic flocculation in the same fashion as the velocity gradients produced by slow mixing in flocculating basins (*6–8*). In the following we analyze this process to determine the additional terms which must be included in the continuity equations to take this effect into account. We then present results for a sludge blanket clarifier run under a variety of conditions with and without inclusion of this mechanism of flocculation.

ANALYSIS

We take for our model an upflow clarifier of the sludge blanket type in which composite particles are formed by flocculation from unit elementary particles or smaller composite particles, disrupted by viscous drag forces, and moved in a vertical direction by the interplay between gravity and viscous drag. The continuity equation for n-particles (composites of n elementary particles) is

$$\frac{\partial c_n}{\partial t}(x, t) = \frac{-\partial}{\partial x}(v_n' c_n) + \frac{\partial}{\partial x}\left(D_n \frac{\partial c_n}{\partial x}\right) + F_n[\mathbf{c}(x, t)], \qquad n = 1, 2, \ldots, N \tag{1}$$

where t = time

c_n = number density of n-particles at (x, t)

x = distance from the bottom of the clarifier

v_n' = velocity in the laboratory frame of reference of an n-particle at (x, t)

D_n = effective diffusion constant of an n-particle at (x, t)

$\mathbf{c}(x, t) = [c_1, c_2, \ldots, c_N]$

F_n = flocculation and disruption terms

N = number of elementary particles in the largest composite particle permitted

$A(x)$ = cross-sectional area of clarifier at a height x above the bottom

l = height of sludge wasting plane

L = height of clarifier

v_k = velocity of a k-particle relative to the surrounding liquid

V_1 = volume of an elementary particle

V_k = volume of a k-particle, $= kV_1$

C = volume fraction solids, $= \sum_{n=1}^{N} c_n(x, t) V_n$

$v''(x, t)$ = velocity of liquid at (x, t) relative to the laboratory

In our previous work we calculated the contributions from flocculation and floc disruption to be given by

$$\begin{aligned} F_n = {} & \sum_{j=1}^{[n/2]} c_j c_{n-j} |v_j - v_{n-j}| \pi (r_j + r_{n-j})^2 - \sum_{j=1}^{N-n} c_j c_n |v_j - v_n| \pi (r_j + r_n)^2 \\ & + \sum_{j=n+1}^{N} k_{n,j-n}^{j} c_j (1 + \delta_{n,j-n}) - \sum_{j=1}^{[n/2]} k_{j,n-j}^{n} c_j \end{aligned} \tag{2}$$

where $\delta_{ij} = 0$ if $i \neq j$; $= 1$ if $i = j$

$[n/2]$ = largest integer $\leq n/2$

Viscosity we take to be given by (*9*)

$$\eta = \eta_0 \exp\left[\frac{2.5C + 2.7C^2}{1 - 0.609C}\right] \tag{3}$$

where η_0 is the viscosity of the pure liquid. Slurry density is given by

$$\rho_{sl} = \rho_s C + \rho_l(1 - C) \tag{4}$$

where ρ_s is the solid density and ρ_l is the liquid density. We define

$$\Delta\rho = \rho_s - \rho_{sl} = (\rho_s - \rho_l)(1 - C) \tag{5}$$

We take the velocity of a k-particle relative to the surrounding liquid to be given by *(10)*

$$-v_k = u_k = \frac{2g(\Delta\rho)r_k^2}{9\eta\left[1 + \frac{1}{4}\left(\frac{\rho_{sl}r_k u_k}{2\eta}\right)^{1/2} + 0.34\rho_{sl}r_k u_k\right]} \tag{6}$$

Here g is the gravitational constant. The velocity of a k-particle relative to the laboratory is given by

$$v_k' = v_k - \sum_{n=1}^{N} v_n c_n V_n + \frac{Q_{\text{feed}}}{A(x)} \tag{7}$$

below the sludge wasting plane, and by

$$v_k' = v_k - \sum_{n=1}^{N} v_n c_n V_n + \frac{Q_{\text{feed}} - Q_{\text{waste}}}{A(x)} \tag{8}$$

above this plane. Here Q_{feed} = volumetric feed rate and Q_{waste} = volumetric rate of sludge withdrawal. As before, we take

$$k_{n,j-n}^{j} = \frac{\kappa j j!}{n!(j-n)!} \frac{[N/2]!(N - [N/2])!}{NN!} \tag{9}$$

where κ is a proportionality constant.

We wish to examine the effect of the energy dissipation of the falling floc particles in creating small-eddy turbulence, thereby increasing floc-floc collision frequencies and the rate of flocculation. We analyze the quiescent case. The power dissipation per unit volume is given by

$$\varepsilon = -g\rho_s \sum_n c_n(x, t)V_n v_n'(x, t) - g\rho_l\left(1 - \sum_n c_n V_n\right)v''(x, t) \tag{10}$$

We note that for the quiescent case

$$v'' = \sum_n v_n c_n V_n = \text{velocity of liquid relative to the laboratory} \tag{11}$$

and

$$v_k' = v_k - \sum_n v_n c_n V_n = \text{velocity of } k\text{-particle relative to the laboratory} \tag{12}$$

and substitute these results in Eq. (10) to obtain, after simplification,

$$\varepsilon = -g(\rho_s - \rho_l)(\sum_n v_n c_n V_n)(1 - \sum_n c_n V_n) \tag{13}$$

for the power dissipation per unit volume. We calculate a root-mean-square velocity gradient by the method of Camp and Stein (*11*),

$$G = (\varepsilon/\eta)^{1/2} \tag{14}$$

and, from this, again following Camp and Stein, we find that the collision frequency per unit volume of *i*-particles with *j*-particles is given as

$$z_{ij} = \frac{4}{3} c_i c_j (r_i + r_j)^3 G\alpha \left(1 - \frac{\delta_{ij}}{2}\right) \tag{15}$$

We introduce a factor $\alpha <$ unity into Eq. (15) to take into account the fact that the eddies generated by these falling particles will be comparable in size to the particles, and that these eddies will be less efficient than larger eddies in inducing collisions to take place. This point is discussed in some detail by Argaman and Kaufman (*5*).

We therefore add the following terms to Eq. (2) to take into account the flocculation due to turbulence-induced collisions:

$$F'_n = \sum_{j=1}^{[n/2]} \frac{4}{3} \alpha G (r_j + r_{n-j})^3 c_j c_{n-j} \left(1 - \frac{\delta_{j,n-j}}{2}\right) - \sum_{j=1}^{N-n} \frac{4}{3} \alpha G (r_j + r_n)^3 c_j c_n \tag{16}$$

The boundary layers of our particles are of the scale of the particles themselves; we therefore expect that α should be substantially less than unity, and we rather arbitrarily set $\alpha = 0.5$.

Note that G, through Eqs. (13) and (14), is roughly proportional to $C^{1/2}$, provided C is not too large. This means that F'_n provides a mechanism for flocculating particles which varies roughly as the 5/2 power of the solids volume fraction. This turbulence flocculation mechanism also allows the coagulation of particles which are identical in size, which is not permitted by the catch-up mechanism described by the first two sums in Eq. (2).

We solve Eq. (1) by numerical integration, using the same methods employed previously (*3*); we simply add the terms exhibited in Eq. (16) to Eqs. (10)–(13) of Ref. *3* and proceed exactly as before.

RESULTS, SLUDGE BLANKET CLARIFIERS

The sludge blanket clarifier for which computations were carried out is diagrammed in Fig. 1. Influent feed is at the bottom, sludge is wasted

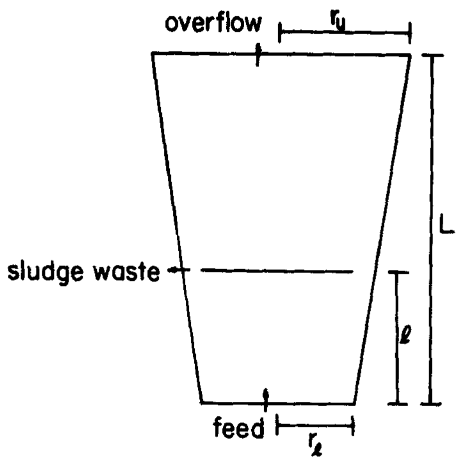

FIG. 1. Sludge blanket clarifier. L = 100 cm, l = 47.5 cm, r_u = 50 cm, r_l = 30 cm, Q_{waste} = 200 ml/sec; η_0 = 0.01 P, ρ_s = 1.05 g/cm^3, ρ_l = 1.00 g/cm^3, influent solids volume fraction = 0.002, N = 4, I = 20 in all runs. All runs start with the clarifier filled with slurry. κ=0.01 sec^{-1}, $c^\circ(n)$=const/n^2, r_1 = 0.02 cm.

from a plane somewhere in the middle of the clarifier, and effluent is overflowed from the top. Clarifier parameters and slurry characteristics are given in the caption to Fig. 1. Figure 2 compares the percent solids removal (measured by sludge solids volume fraction) for runs at various feed rates made with and without inclusion of the turbulence flocculation mechanism. We find that percent removal (by SSVF) is somewhat lower if the turbulence flocculation mechanism is excluded. These figures were all obtained after the clarifier had been in operation for 3000 sec.

Figure 3 shows plots of percent removal (SSVF) as functions of time at a feed rate of 2500 ml/sec with and without turbulence flocculation. Marked differences in the way the two systems approach a steady state are apparent, with the curve for the system without turbulence flocculation exhibiting quite a bit more structure and a slower approach to the steady state than the system with turbulence flocculation.

In Fig. 4 we see similar plots at a feed rate of 3000 ml/sec. The system with turbulence flocculation exhibits 100% solids removal (SSVF) after 1900 sec. The system without has not reached 90% solids removal (SSVF) at 3000 sec, and is in fact slowly building up a high concentration of fines in the upper section of the clarifier which cannot coagulate and will eventually pass out in the clarifier overflow.

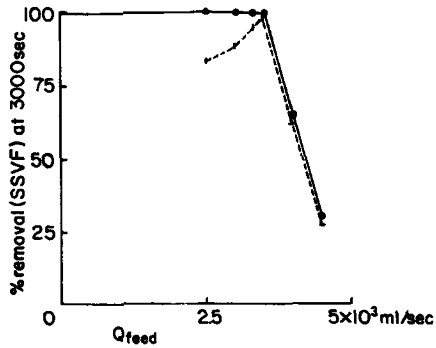

FIG. 2. Percent solids removal (by sludge solids volume fraction, SSVF) vs feed rate: (●) turbulence flocculation included, (- -) turbulence flocculation not included. The data are reported after 3000 sec of simulated operation.

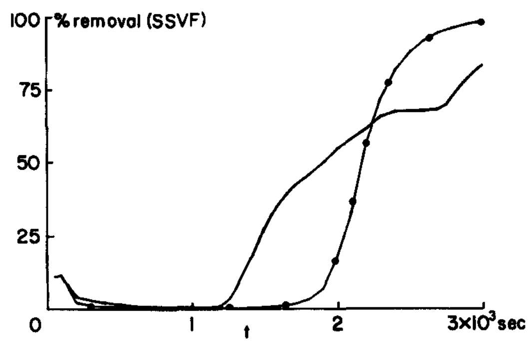

FIG. 3. Percent solids removal (SSVF) vs time. $Q_{feed} = 2500$ ml/sec: (●) turbulence flocculation included, (—) turbulence flocculation not included.

The next three Figures—5, 6, and 7—present similar pairs of plots at feed rates of 3500, 4000, and 4500 ml/sec. In all cases inclusion of the turbulence flocculation mechanism improves the clarifier performance, but the effect decreases as the feed rate increases. Figure 6 shows marked maxima in the plots; these occur as the densest part of the sludge blanket rises through the plane at which sludge wasting takes place.

Figure 8 shows solids volume fractions as functions of position in the clarifier for various feed rates at 3000 sec. Comparison of these results with those of Fig. 2 show that percent solids removal (SSVF) decreases drastically if the densest part of the sludge blanket rises above the sludge

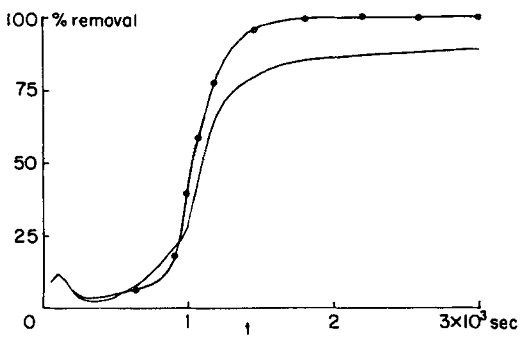

FIG. 4. Percent solids removal (SSVF) vs time. $Q_{feed} = 3000$ ml/sec: (●) turbulence flocculation included, (—) turbulence flocculation not included.

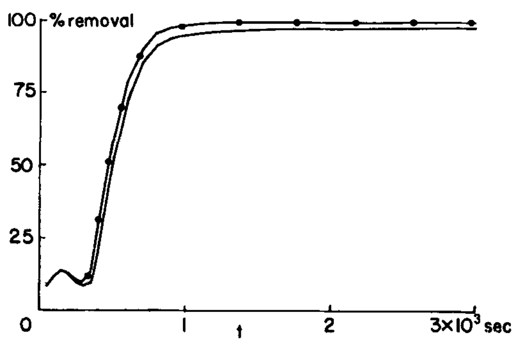

FIG. 5. Percent solids removal (SSVF) vs time. $Q_{feed} = 3500$ ml/sec: (●) turbulence flocculation included, (—) turbulence flocculation not included.

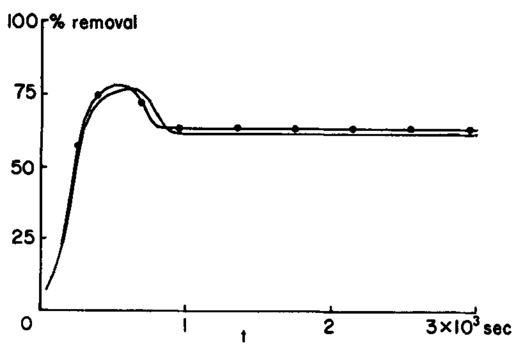

FIG. 6. Percent solids removal (SSVF) vs time. $Q_{feed} = 4000$ ml/sec: (●) turbulence flocculation included, (—) turbulence flocculation not included.

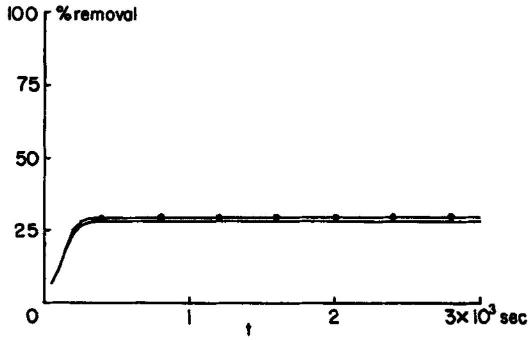

FIG. 7. Percent solids removal (SSVF) vs time. $Q_{feed} = 4500$ ml/sec: (●) turbulence flocculation included, (—) turbulence flocculation not included.

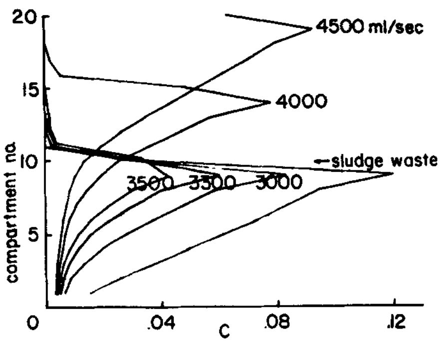

FIG. 8. Solids volume fraction distribution in the clarifier after 3000 sec for various feed rates. Turbulence flocculation is included. Steady-state solids removal efficiencies for these runs are estimated to be 100% at feed rates of 2500 and 3000, 99.57% at 3300, 99.62% at 3500, 63.32% at 4000, and 29.32% at 4500 ml/sec.

wasting plane. Evidently one would be well-advised to locate a solids measuring device a short distance above the sludge waste plane and use its output to control the feed rate to the clarifier.

Figures 9, 10, and 11 compare solids volume fraction profiles in the clarifier with and without turbulence flocculation for feed rates of 2500, 3000, and 3500 ml/sec. In all three cases the system without turbulence flocculation exhibits secondary sludge blankets. A trace of such a secondary blanket is seen for the system with turbulence flocculation at a feed rate of 3500 ml/sec; no secondary blankets are observed with turbulence

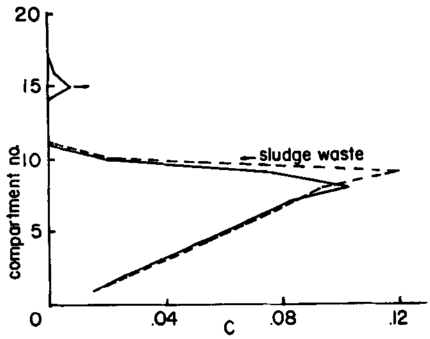

FIG. 9. Comparison of solids volume fraction profiles at 3000 sec, $Q_{feed} = 2500$ ml/sec: (- -) turbulence flocculation included, (—) turbulence flocculation not included.

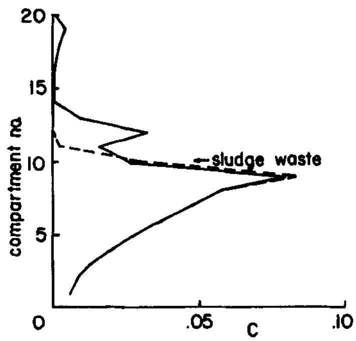

FIG. 10. Comparison of solids volume fraction profiles at 3000 sec, $Q_{feed} = 3000$ ml/sec: (- -) turbulence flocculation included, (—) turbulence flocculation not included.

flocculation at lower flow rates. The presence or absence of these secondary blankets in sludge blanket clarifiers appears to provide an experimental test of the correctness of our turbulence flocculation mechanism.

We note that the eventual development of a steady state in these clarifiers when fed at a constant rate is by no means a foregone conclusion. The differential equations are strongly coupled and quite nonlinear, raising the possibility of quasi-periodic solutions (so-called limit cycles). We searched assiduously for such behavior, but did not find it in our simulations of these sludge blanket clarifiers. Approach to a steady state might be very slow, especially at low flow rates, but we saw no

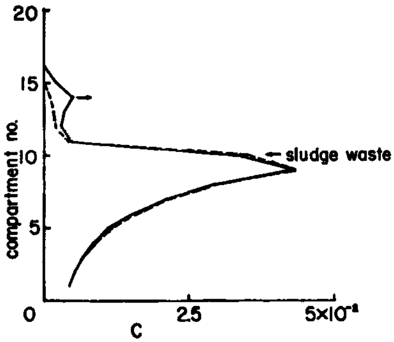

FIG. 11. Comparison of solids volume fraction profiles at 3000 sec, Q_{feed} = 3500 ml/sec: (- -) turbulence flocculation included, (—) turbulence flocculation not included.

indications on any of the runs that a steady state was not being approached.

ANCILLARY EQUIPMENT

One of the numerous areas which can be explored with various modifications of these clarifier simulators is the effect of putting an equalizing tank in the clarifier influent line to reduce shock loads in solids concentration and/or feed rate when these quantities are permitted to vary with time. We examine the output feed rate Q_o and output solids concentration from a stirred tank flow stabilizer having an input feed rate $Q_i(t)$ and input solids concentration $c_i(t)$. We let $V(t)$ be the volume of liquid in the tank at time t, and assume that $Q_o = Q_o(V)$ is a known function of V.

We then obtain

$$\dot{V} = Q_i(t) - Q_o(V) \tag{17}$$

and

$$\frac{d}{dt}[V(t)c_o(t)] = Q_i(t)c_i(t) - Q_o(V)c_o(t) \tag{18}$$

With the use of Eq. (17), Eq. (18) can be simplified to yield

$$\dot{c}_o + \frac{Q_i(t)}{V(t)}c_o = \frac{Q_i}{V}c_i(t) \tag{19}$$

We solve Eqs. (17) and (19) simultaneously for $V(t)$ and $c_o(t)$ in terms of

$V(0)$, $Q_i(t)$, and $c_i(t)$ by numerical integration. From $V(t)$ we then calculate $Q_o(t)$. $Q_o(t)$ and $c_o(t)$ are the input to the clarifier, and are used in the clarifier simulator in the usual way.

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